Upwind based High Resolution Shock Capturing Schemes for Hyperbolic Conservation Laws

ABSTRACT

In this thesis, we construct, analyse and implement upwind based high resolution shock capturing schemes for hyperbolic conservation laws. Construction of these high resolution schemes is based on adding anti-diffusive flux function to the diffusive numerical flux function of any first order accurate scheme through the flux limiter function. The reason behind using upwinding is that upwind based schemes are capable to give better numerical approximation to the hyperbolic conservation laws because they inherit the property of satisfying physical hyperbolicity condition associated with these equations.

We first construct the high resolution scheme for linear hyperbolic conservation laws and related convectively dominated diffusion equations followed by its extension to linear system using wave splitting. A general procedure for constructing semi-discrete high resolution schemes for nonlinear problems using numerical flux function of any low order entropy satisfying scheme along with the numerical flux of nonlinear second order accurate upwind scheme with flux limiters is presented. We also give an efficient high resolution relaxation scheme for general hyperbolic system of conservation laws using the a framework which converts the nonlinear conservation law into a linear system of conservation laws with nonlinear source term. We also present a numerical study for inhomogeneous problems. The constructed schemes are implemented of various benchmark test problems which validate the theoretical results. The numerical results not only give high resolution with higher accuracy but also do not introduce unphysical oscillations to the numerical solution. For nonlinear case they produce correct physical solution.